ANALYTIC NUMBER THEORY IN FUNCTION FIELDS
TCC 2015
PROBLEM SHEET 3

JULIO ANDRADE

1-) Suppose h is a complex valued function on monics in A and that the limit as n tends to infinity of $\mathrm{Ave}_n(h)$ is equatl to α. Show

$$\lim_{n\to\infty}(1+q+q^2+\cdots+q^n)^{-1}\sum_{\substack{f \text{ monic}\\ \deg(f)\leq n}} h(f)=\alpha.$$

2-) Let $\mu(m)$ be the Möbius function on monic polynomials which we have introduced in the lectures. Consider the sum $\sum_{\deg(m)=n}\mu(m)$ over monic polynomials of degree n. Show the value of this sum is 1 if $n=0$, $-q$ if $n=1$, and 0 if $n>1$.

3-) For each integer $k\geq 1$ define $\sigma_k(m)=\sum_{f|m}|f|^k$. Calculate $\mathrm{Ave}_n(\sigma_k)$.

4-) Consider the von-Mangoldt function in A. Show that

$$\sum_{f|m}\Lambda(f)=\log_q|m|.$$

5-) Show that

$$D_\Lambda(s)=-\zeta_A^{'}(s)/\zeta_A(s).$$

Use this to evaluate $\sum_{\deg(m)=n}\Lambda(m)$.

6-) Recall that $d(m)$ is the number of monic divisors of m. Show

$$\sum_{m \text{ monic}}\frac{d(m)^2}{|m|^s}=\frac{\zeta_A(s)^4}{\zeta_A(2s)}.$$

Use this to evaluate $\sum_{\deg(m)=n}d(m)^2$.

Date: April 28, 2015.

University of Oxford - Mathematical Institute
E-mail address: j.c.andrade.math@gmail.com