ANALYTIC NUMBER THEORY IN FUNCTION FIELDS
TCC 2015
PROBLEM SHEET 8

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1-) Let M be even and positve. And consider that the following sum is over all non-square monic polynomials of degree M in A. Also suppose that $s \neq \frac{1}{2}$ or 1. Prove that

$$q^{-M} \sum L(s, \chi_m) = \frac{\zeta_A(2s)}{\zeta_A(2s+1)} - \left(1 - \frac{1}{q}\right) (q^{1-2s})^{\frac{M}{2}} \zeta_A(2s) - q^{-\frac{M}{2}} \left(\frac{\zeta_A(2s)}{\zeta_A(2s+1)} - \left(1 - \frac{1}{q}\right) (q^{1-s})^M \zeta_A(s) \right).$$

2-) Let M be even and positve. And consider that the following sum is over all non-square monic polynomials of degree M in A. Also suppose that $s = 1$. Prove that

$$q^{-M} \sum L(1, \chi_m) = \frac{\zeta_A(2)}{\zeta_A(3)} - q^{-\frac{M}{2}} \left(2 + \left(1 - \frac{1}{q}\right)(M-1) \right).$$

3-) Let M be positive and even, and let $\gamma \in \mathbb{F}^*$ be a non-square constant. The following sum is over all non-square monic polynomials of degree M. For $s \neq \frac{1}{2}$ prove that

$$q^{-M} \sum L(s, \chi_{\gamma m}) = \frac{\zeta_A(2s)}{\zeta_A(2s+1)} - \left(1 - \frac{1}{q}\right) (q^{1-2s})^{\frac{M}{2}} \zeta_A(2s) - q^{-\frac{M}{2}} \left(\frac{1 + q^{-s}}{1 + q^{1-s}} - \left(1 - \frac{1}{q}\right) \frac{(q^{1-s})^M}{1 + q^{1-s}} \right).$$

4-) In the situations of problems 1,2 and 3 find a formula for

$$q^{-M} \sum_{\substack{m \text{ monic} \\ \deg(m) = M}} L(\tfrac{1}{2}, \chi_m).$$

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