Number Theory and Function Fields at the Crossroads 2016
Focused Research Workshop
January 20 - 22, 2016
University of Exeter

1. Schedule of Talks

Wednesday, 20/01/2016

9:00 - 9:30: **Arrival Coffee/Tea**
9:30 - 10:30: **Chris Hughes**
10:30 - 11:00: **Coffee/Tea**
11:00 - 11:30: **Alexandra Florea**
11:30 - 12:00: **Edva Roditty-Gershon**
12:00 - 12:30: **Dan Carmon**
12:30 - 13:30: **Lunch** (Reed Woodbridge Suite - Reed Hall)
14:00 - 15:00: **Chantal David**
15:00 - 15:30: **Coffee/Tea**
15:30 - 18:00: **Open Discussion and Brainstorming**

Thursday, 21/01/2016

9:00 - 9:30: **Arrival Coffee/Tea**
9:30 - 10:30: **Brian Conrey**
10:30 - 11:00: **Coffee/Tea**
11:00 - 11:30: **Adam Harper**
11:30 - 12:00: **Alexei Entin**
12:00 - 12:30: **Brad Rodgers**
12:30 - 13:30: **Lunch** (Reed Woodbridge Suite - Reed Hall)
14:00 - 15:00: **Andrew Granville**
15:00 - 15:30: **Coffee/Tea**
15:30 - 18:00: **Open Discussion and Brainstorming**
19:30 - 23:00: **Dinner** (Reed Woodbridge Suite - Reed Hall)

Friday, 22/01/2016

9:00 - 9:30: **Arrival Coffee/Tea**
9:30 - 10:30: **Tim Browning**
10:30 - 11:00: **Coffee/Tea**
11:00 - 12:00: **Lior Bary-Soroker**
12:00 - 12:30: **Min Lee**
12:30 - 13:30: **Lunch** (Reed Woodbridge Suite - Reed Hall)
14:00 - 15:00: **Open Discussion and Brainstorming**
15:00 - 15:30: **Coffee/Tea**
15:30 - 18:00: **Open Discussion and Brainstorming**

2. Titles and Abstract

Lior Bary-Soroker, Tel-Aviv University, Tel-Aviv

Title: *Statistics of arithmetic functions in function fields*

Abstract: Some of the deepest theorems and conjectures in number theory may be viewed as certain statistical properties of certain arithmetic functions. Taking as an example the von Mongoldt arithmetic function: computing its mean value is equivalent to the Prime Number Theorem and computing its autocorrelations is equivalent to the Hardy-Littlewood prime tuple conjecture (which is more general than the twin prime conjecture).

Problems of this nature tend to be extremely difficult, and many times are considered as completely out of reach. However, the situation in function fields is different. After a series of works by a series of authors in the last five years, we now in the position that we completely understand the statistics of arithmetic functions in function fields in the large finite field limit for arithmetic functions that depends only on the "factorization type". This set of functions includes the von Mongoldt function, the Mobius function, the divisor functions, etc.

The objective of this talk is to discuss the series of works that led to this understanding, and to present a new result that enlarge the family of arithmetic functions for which these results hold. This new family includes, as a typical example, the indicator function of the analogue of sums of two squares.

Tim Browning, University of Bristol, Bristol

Title: *Algebraic geometry over finite fields via counting*

Abstract: One of the big open problems in algebraic geometry is whether or not every Fano hypersurface is unirational. A starting point for answering this question is to study the space of rational curves on hypersurfaces. We show how the Hardy-Littlewood circle method can help with this programme in the setting of finite fields.

Dan Carmon, Tel-Aviv University, Tel-Aviv

Title: *Square-free values of polynomials with large coefficients*

Abstract: How many numbers between X and $X + H$ are square-free, where X is large and $H > X^{\epsilon}$? In how many ways can a large number N be given as a sum $N = x^k + r$ of a positive k-th power and a positive square-free number r? In full generality, both questions are still mostly open. They can be seen as special cases of a more general question - how many values of a polynomial $f(x)$ are square-free, where the coefficients of the polynomial are much larger than the values the argument assumes? We answer these questions in the function field setting, over a fixed finite field with degrees going to infinity, following the techniques of Poonen and Lando, who solved similar questions for polynomials with fixed coefficients.

Brian Conrey, AIM and University of Bristol, California

Title: *Approaches to RH*

Abstract: We recall some of the classical approaches to the Riemann Hypothesis in the hopes that one of them could be used to give a new proof in the function field case.

Chantal David, Concordia University, Montréal

Title: *Statistics for cyclic covers over finite fields*

Abstract: We study the fluctuations in the number of points in families of curves of genus g over the finite field $\mathbb{F}_q$. There are 2 types of limiting distribution, depending on the limit which is taken. When the genus is fixed and q tends to ∞, the statistics are given by statistics on random matrices in a symmetry group determined by the monodromy group of the family by the equidistribution theorems of Deligne and Katz-Sarnak. When q is fixed

and the genus tends to infinity, the distribution is given by a a natural probabilistic model, in terms of a sum of $q+1$ independent identically distributed random variables.

The case of hyperelliptic curves $Y^2 = F(X)$ of genus g was considered by Kurlberg and Rudnick, and the case of cyclic covers of prime degree l with model $Y^l = F(X)$ Bucur, David, Feigon and Lalin. We present in this talk the results for the general case $Y^n = F(X)$, where n is not necessarily a prime. This is the work of my Ph.D. student Patrick Meisner.

Alexandra Florea, Stanford University, Stanford

Title: *Mean value of quadratic Dirichlet L-functions in function fields*

Abstract: We discuss moments of L-functions in function fields, in the hyperelliptic ensemble, focusing on the first moment of this family of L-functions. Summing $L(1/2, \chi_D)$ over monic, square-free polynomials D of degree $2g+1$ (with q fixed and $g \to \infty$), Andrade and Keating obtained an asymptotic formula with a main term of size $|D| \log_q |D|$. We will describe a different approach which allows us to compute a secondary term of size $|D|^{1/3} \log_q |D|$.

Andrew Granville, University College London and Université de Montréal, London

Title: *Sieving differences*

Abstract: While investigating why the Maynard-Tao sieve weights work so well, we stumbled upon some big differences between their large values distribution and that of their analogy over finite fields. This is joint work with Dimitris Koukoulopoulos and James Maynard.

Adam Harper, University of Cambridge, Cambridge

Title: *Multiplicative functions in function fields*

Abstract: Halász's theorem is a classical result giving a "lossless" upper bound for the mean value of a multiplicative function over the integers. In forthcoming work with A. Granville and K. Soundararajan, we give a (hopefully) fairly simple proof of Halász's theorem, and show how this generalises to more complicated settings like short intervals and arithmetic progressions. In this talk I will try to present a version of our proof for multiplicative functions over function fields (i.e. multiplicative functions defined on polynomials). For the most part, this setting helps to simplify the analysis compared with the integer setting. However, there are also some kinds of behaviour that one doesn't see over the integers, which I will comment on if time allows.

Chris Hughes, University of York, York

Title: *Modelling large values of L-functions*

Abstract: Understanding the extreme values of the Riemann zeta function and other L-functions has long been a challenge. This talk will survey what's known and what's conjectured, and how the different models elucidate different aspects of the problem.

Bradley Rodgers, University of Michigan, Michigan

Title: *Explicit formulas and symmetric function theory*

Abstract: In this talk I will attempt to outline some recent results regarding the variance of sums of arithmetic functions over random short intervals in $\mathbb{F}_q[T]$ and explain a connection between these results and the distribution of cycle types in the symmetric group.

Edva Roditty-Gershon, University of Bristol, Bristol

Title: *Arithmetic Statistics in Function Fields*

Abstract: One of the most famous conjectures in number theory is the Hardy-Littlewood conjecture, which gives an asymptotic for the number of integers n up to X such that for a given tuple of integers $a_1, .., a_k$ all the numbers $n+a_1, .., n+a_k$ are prime. This quantifies and generalises the twin-prime conjecture. Function field analogue of this problem has recently been resolved in the limit of large finite field size q by Lior Bary-Soroker. However, in this limit the correlations disappear: the arithmetic functions become uncorrelated. It is therefore important to understand the terms of lower order in q, which must account for the correlations. We compute averages of these terms which detect correlations. Our results show that there is considerable cancellation in the averaging and have implications for the rate at which correlations disappear when q tends to infinity. This is a joint work with Jon Keating

3. Participants

Ardavan Afshar (University College London)

Julio Andrade (University of Exeter and University of Oxford)

Lior Bary-Soroker (Tel-Aviv University)

Tim Browning (University of Bristol)

Hung Bui (University of Manchester)

Nigel Byott (University of Exeter)

Dan Carmon (Tel-Aviv University)

Brian Conrey (AIM)

Chantal David (Concordia University)

Alexei Entin (Stanford University)

Alexandra Florea (Stanford University)

Andrew Granville (University College London and Université de Montréal)

Adam Harper (University of Cambridge)

Chris Hughes (University of York)

Henri Johnston (University of Exeter)

Jon Keating (University of Bristol)

Oleksiy Klurman (University College London)

Min Lee (University of Bristol)

Adelina Manzateanu (University of Bristol)

Brad Rodgers (University of Michigan)

Edva Roditty-Gershon (University of Bristol)

Igor Wigman (King's College)

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