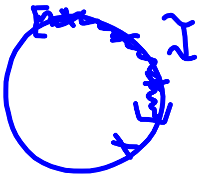
$$\zeta_{K/\mathbb{F}_q t}(s) = \frac{L_K(u)}{(1-u)(1-qu)},$$

$\underline{\underline{L_K(u)}} \in \mathbb{Z}[u]$, $\deg L_K = 2g$



$g \to \infty$

Katz-Sarnak (g-fix, $q \to \infty$)

Thm (Katz-Sarnak): F continuous in $\{\Theta_C, C \in \mathcal{H}_{q,2g+2}\}$. $\boxed{g\text{-fix}}$

$$\lim_{q \to \infty} \frac{1}{\# \mathcal{H}_{q,2g+2}} \sum_{C \in \mathcal{H}_{q,2g+2}} F(\Theta_C) = \int_{USp(2g)} F(U)\,dU$$

$$\mathcal{H}_{2g+2,q} = \{Q \in \mathbb{F}_q[x],\ \text{monic},\ \deg Q = 2g+2,\ Q \text{ sq.-free}\}$$

$$C \in \mathcal{H}_{2g+2}, \quad C/\mathbb{F}_q \quad C: y^2 = Q(x)$$

$D \in \mathcal{H}_{2g+2}$

$$\chi_D(P) = \left(\frac{D}{P}\right) = \begin{cases} -1 & P \nmid D \quad D \neq \square(P \\ 0 & P \mid D \\ 1 & P \nmid D \quad D = \square(P) \end{cases}$$

$\chi_D(f) =$

$f = P_1^{\alpha_1} \dots P_r^{\alpha_r}$

$$\begin{tikzcd}
Q \in \mathcal{H}_{2g+2} \arrow[r] & C/\mathbb{F}_q \arrow[r] & Z_{C_Q}(u) \arrow[d] \\
& L^*(s, \chi_Q) & P_{C_Q}(u) \arrow[l]
\end{tikzcd}$$

$$\lim_{g\to\infty} \mathrm{Prob}_{\mathcal{H}_{2g+2}}\left(\cdots\right)$$

$$\frac{1}{\#\mathcal{H}_{2g+2}} \sum_{D\in\mathcal{H}} F(D)$$

$$\chi_Q(f) \xrightarrow{f=\square} \frac{1}{\#\mathcal{Q}} \sum_{Q \in \mathcal{Q}} \chi_Q(f=\square) = 1 + O(\ldots)$$

$$\chi_Q(f) \xrightarrow{f \neq \square} \frac{1}{\#\mathcal{Q}} \sum_{Q \in \mathcal{Q}}{}' \chi_Q(f) \ll 2^{\deg f} q^{g}$$